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Descriptions and group reference*

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Recent work on definite descriptions, inspired primarily by Strawson and Donnellan, has provided many insights into the complexity and diversity of this linguistic phenomenon. These insights, in turn, have raised growing doubts about the availability of Russell's theory of descriptions for a large number of cases without supplementing, as yet, Russell's theory with some alternative theory. What is suggested instead are distinctions, classifications, taxonomies and the like. A persistent problem, then, is to understand 'why the same construction with a definite article is used for a wide variety of cases'.

In this paper I shall suggest some steps towards a unified account of a wide variety of uses of English definite descriptions without imposing any arbitrary restrictions on the phenomenal domain.² Details of this account apart, I shall try to develop two general claims. First, the critiques of Russell's theory of descriptions developed by Strawson and Donnellan are in the right direction, but neither of them go far enough. The limitations with their critiques, in my view, are a result of their continued, if only partial, adherence to Russell's theory. Russell's theory, I shall argue, has nothing at all to do with the uses of the English definite article.³ Russell's theory probably applies to the idiom of "exactly one", but I would not go into that. Secondly, a complete dissociation from Russell's theory brings into sharp focus a fundamental communicative function of the English definite article, a function which was only dimly appreciated before.⁴ The definite article, I shall argue, helps establish, in a fairly regular way, a guarantee of communication between a speaker and an audience such that they can proceed to further exchange of information.

My methodology would be as follows. First, I shall try to bring out, as clearly as I can, just why Russell's theory cannot be applied to certain cases either ignored or unnoticed by Russell (Section I). This stage of the discussion, then, should generate at least a loose dichotomy of the uses of definite descriptions—those that do not obey Russell's theory (Section II) and those that supposedly do (Section III). Secondly, in a gradual reversal of motivation, I shall dilute the dichotomy by placing the supposedly dichotomous phenomena on a (intuitive) linear scale (Section IV). Next, I shall furnish some preliminary account of how such a linear scale is 'continuously' generated by certain underlying factors governing uses of definite descriptions (Section V). Finally, I shall touch on some problems of misdescriptions (Section VI) and vacuous descriptions (Section VII) which led to the philosophical discussions of definite descriptions in the first place.

According to Russell's theory of descriptions, an English sentence of the form "The G is F" is to be analysed, at least, as "One and only one thing Gs and that thing also Fs". Following Kripke, we may think of this theory as a theory to thing also 1's . I only to the effect that English is a 'Weak Russell Language'. The theory does not deny that "The G" is a 'primitive designator'. The only conditions that the theory imposes on sentences of the form "The G is F" are: (i) the referent of "The G" is that unique object which satisfies the description "G"; and (ii) the sentence is true if the predicate "F" is true of the referent of "The G", the sentence is false otherwise. Russell's original theory may be construed as a theory to the effect that English is a 'Strong Russell Language' where "The G" is no longer a 'primitive designator' but an 'incomplete symbol'. Strawson and Kaplan, among others, have raised pursuasive objections against the strong theory from various directions. It is generally believed that such objections leave the weak theory pretty well untouched except, of course, the vacuous description problem. If "The G" is a 'primitive designator' then, if "The G" is vacuous, both "The G is F" and "The G is not F", according to the Weak Theory, are false. Since Russell's scope distinctions are not available for the Weak Theory, this would lead to a violation of the law of excluded middle. My primary concern in this paper, however, is with condition (i) in the statement of Weak Theory; in particular, I am concerned with the notion of 'an object uniquely satisfying the description G'. This notion, surely, is involved in any Russellian Theory. If this notion is found to be inadequate, there is no need for moving on to the Strong Theory. The need, then, would be for an alternative non-Russellian theory which can handle the vacuous description problem.9 My discussion, therefore, will be confined to the Weak Theory.10

A first problem with Russell's theory is that it applies, if at all, only to phrases of the form "The G" in the singular. The theory, thus, ignores phrases like "The brothers of ...", "The numbers preceding ...". If we follow the 'quantificational' interpretation, then perhaps it is possible to generalize Russell's interpretation in various ways to include phrases in the plural. According to Chomsky, 11 for example, we can think of an use of the definite article "The" as signifying a universal quantification with existential import and the semantics of such a quantification may be captured in standard set theory. I cannot discuss here whether, given our interest in the Weak Theory, a Chomskian solution to this plural description problem meshes with putative solutions to the further problems raised below. If my ultimate account of definite descriptions across the board is correct, we do not need a solution along Chomskian lines.

A second, more fundamental, problem has been raised by Keith Donnellan.¹² According to Donnellan, a definite description, say, "The President", may be used attributively to denote whoever happens to be the President as

in "The President has lived in the White House since 1800"; or, the same phrase may be used referentially to pick out (or to enable one's audience to pick out) a particular President as in "The President has been married since 1945". 13 Donnellan has shown further that not only the same phrase but even the same sentence may, on different occasions, be used differently ("The murderer of Smith is insane"). Moreover, in a referential use, an individual may be successfully picked out by the audience, even if the individual does not (fully?) fit the description. Donnellan complains that Russell's theory explains, at most, the attributive uses of definite descriptions, the uses based on general, non-contextual knowledge; the uses in which the only way of successful reference/denotation is via a (perfect?) fit with the description.

Saul Kripke, ¹⁴ among others, has suggested, following Grice, a way of saving Russell's theory from Donnellan's complaint. Russell's theory, for Kripke, concerns the 'semantic referent' of "The G"—the object that uniquely satisfies "G". When using "The G" in a declarative sentence, the user/speaker has a referent 'in mind' which may or may not match the semantic referent. Paradigmatic attributive uses are those where the speaker's referent¹⁵ and the 'semantic referent' are identical. Paradigmatic referential uses are those where speaker's referent is (almost) totally divorced from the semantic referent. If the audience has access, somehow, to the speaker's referent, the reference can still be successful despite misdescription. Donnellan's distinction reflects vagaries of speech-act, not ambiguities of language, the Weak Russell Language, in particular. ¹⁶

How successful is this line of reasoning? Suppose, we grant all of the following: (i) That we clearly understand the notion of a speaker's having a referent 'in mind'; (ii) That speaker's reference is not involved in semantic reference; (iii) That all cases inviting Donnellan's distinction can be explained in terms of Kripke's distinction; (iv) That Kripke's distinction is available anyhow in languages without definite descriptions or in languages containing explicit devices for handling Donnellan's distinctions.

Even then, how does a particular classification scheme, Kripke's, save Russell's theory when this theory is threatened by the very notion of classification? Given that Russell's theory is not initially framed with the semantic/pragmatic (or, for that matter, the language/speech-act) distinctions in the relevant way (these distinctions presumably do not arise in the use of an artificial language, viz. the Weak Russell Language), these distinctions can now save Russell's theory only if there is an underlying theory, compatible with Russell's, that generates these distinctions parametrically, i.e., the theory gives a systematic account of how the elements of a distinction change 'values' under varying account of how the elements of a distinction change 'values' under varying circumstances and, so changing, map onto Donnellan's distinction on some circumstances and, so changing, map onto Donnellan's distinction on some points. In particular, we need a systematic account of how the parameters of speaker's reference and semantic reference interact. I shall suggest one such account, but I am not at all sure that this ensuing account is compatible with Russell's.

In any case, Kripke agrees¹⁷ that certain constructions involving the definite article—'The man", "The table"—so-called 'imperfect definite descriptions', ¹⁸ provide at least a 'tentative stab' about Russell. So a third problem with Russell's theory concerns definite descriptions 'where uniquely specifying conditions are not contained in the description itself'. Kripke hints (but does not develop the suggestion) that the problem with this class of definite descriptions may not be solved in a Russellian way by 'regarding (them) as elliptical with uniquely specified conditions added'; that, maybe, a correct understanding of such phrases has something to do with Donnellan's referential use.

If so, then there is at least one variety of case with which Russell's theory may be directly challenged without recourse to misdescriptions; hence, possibly, without depending too much on the notion of speaker's reference. The challenge is direct, simply because, if Kripke's hint is correct, the semantic reference of "The man" is not Russellian. Kripke¹9 hints that "The man" is more like a rigid designator "That table" than like the non-rigid designator "The teacher of Plato". 20 Apart from the uneasy suggestion that two different constructions in a language—"The table" and "That table"—have identical semantics, 21 the rigid distinction between rigid and non-rigid designators seems to forestall any attempt at placing Russellian and non-Russellian descriptions on the same continuum. I need, therefore, a more flexible distinction at this stage. I shall try to develop, in the next section, Kripke's general point in my own way.

Before I do so, let me list some other varieties of cases where Russell's theory does not strictly apply. Constructions involving mass terms ("The sand is hot") and species terms ("The whale is a mammal") do not quite mesh with the condition of 'one and only one thing'. Again, I expect that the ensuing account would be general enough to cover such cases. In general, I do not think that uses of the definite article have much to do with numbers or quantities as the Russellian picture seems to suggest. Thus, I don't even think that an use of "The G" 'implies', in Strawson's sense of 'implies', that one and only one thing Gs.

Further, I expect the ensuing account to cover, more directly, uses of "The" with a stress: "I bought myself the coat yesterday." They can be used referentially, in the subject position. "The man finally arrived in my life"; or, attributively, "The man will never arrive in my life". Still more puzzling are the phrases which, to use Strawson's vivid phrase, 'grow capital letters', lose the stress and turn into proper names—"The White House". "The", in such cases, does seem to carry a Russellian sense of 'uniqueness' all by itself. Strawson²² accounts for such uses in the following way: 'Such phrases are found in print or in writing when one member of some class of events or things is of quite outstanding interest in a certain society.' I shall try to generalize this idea for all uses of "The" across the board, whether spoken or written.

I shall leave it to the readers to judge whether all these promises are, indeed, delivered at the end.

H

In the second paragraph of his 'On Denoting', 23 Russell explains his interest in the denoting phrases. Denoting phrases allow us to think 'about many things with which we have no acquaintance'. Thus, although we have no acquaintance with the centre of mass of the solar system at the first instant of the twentieth century (Russell's example), we know that the phrase "The centre of mass, etc." denotes unambiguously.

The acquaintance/description distinction brings out a marvellous insight into language use—an insight which is obfuscated by Russell's later, quite unnecessary, forays into an epistemology based on sense-data and related notions. In *The Problems of Philosophy*, for example, Russell used this distinction notoriously to argue that we can know physical objects 'indirectly', even though we are 'directly aware' only of sense-data. All of this seems to me to be quite unwarranted; the acquaintance/description distinction makes sense within our ordinary, 'topic-neutral', ways of speaking. Thus, all of you are acquainted with me right now as I speak²⁴—you can say "That person over there". You can as well talk about me in my absence, as "The first speaker on, etc.".

Still, the distinction is important. Given a certain twist in terminology, the importance is nicely captured recently by Barwise and Perry: "Definite descriptions seem to give us a 'further reach' than indexical expressions, a reach that allows us to pluck objects from all manner of resource situations". Thus, in your current 'resource situation' in which I am, say, perceptually accessible, a use of an indexical "That person over there" suffices. You would want to use a definite description "The first speaker on, etc." in a 'resource situation' in which I am no longer so accessible. In traditional terms, definite descriptions allow singular denotation much beyond ostension and, thus, have a 'further reach'. These, then, are Russell's conditions on definite descriptions: definite descriptions allow both singular denotation and a 'further reach' when used non-vacuously. These plus the 'uniqueness condition'.

Notice that, so far, Russell has not placed any other constraint on "G" except that it must be in the singular. So, maybe, Russell's theory extends to the denoting phrase "The kite", as in "The kite is black", which can be used by a native speaker to make a true statement. However, according to Russell: "the, when it is strictly used, involves uniqueness... when we say 'X was the "the, when it is strictly used, involves uniqueness... when we say 'X was the father of Charles II' we not only assert that X had a certain relation to Charles father of Charles II' we not only assert that X had a certain relation to Charles II, but also that nothing else had that relation..." We may think of the uniqueness condition as settling to the satisfaction of the language community, prior to the making of the assertion, a definite answer to the question, nity, prior to the making of the assertion, a definite answer to the question, though we sometimes say "The son of so-and-so" even when so argues, although we sometimes say "The son of so-and-so" even when so

and-so has several sons, it would be *more correct* to say "A son of so-and-so". Similarly, knowing that there are millions of kites in the world, we should never say "The kite is black"; we should say "A kite is black".

Construed as a proposal for language-reform, this is simply intolerable. When I want to say "The kite is black", I want to say precisely that; if I wanted to say "A kite is black", I would have said so. Moreover, this proposal seems all the more absurd, if we remember that for Russell "a kite" means the same as "some kite". 26 Otherwise, according to Russell, if I insist on the 'strict' interpretation of "The" in saying "The kite is black", I would always be saying something false. So the problem is this: although we can think of many commonplace 'resource situations' in which an English speaker would correctly and successfully say "The kite is black", he would be saying something false if by "The kite is black" he means 'one and only one kite is black'. Russellian truth-conditions do not match usage.

The usual moves in this situation, Russellian or otherwise (i.e., moves other than the proposal for language-reform), consist, in general, in a further specification of contexts to which the 'one and only one' part of the assertion must be relativized. A Russellian move would consist in 'eternalizing' an elliptical use of "The table" as, say, "The table in the far left hand corner in room 363 of Hagey Hall at the first instant of 1985", and then giving a Russellian analysis of this supposedly 'eternalized' sentence. As Wettstein has shown, there are indefinitely many ways in which "The table" may be 'eternalized', any of which or even none of which might be the appropriate semantic reference of 'The table'. Nothing much is gained in the attempt to build the context of utterance within the 'eternal' sentence itself. Thus, a non-Russellian move would relativize an utterance of "The table", from the outside, with respect to an ordered n-tuple of contextual co-ordinates for world, time, place, etc. We may even introduce an explicit demonstrative operator ('Dthat' ['The table"]) to handle various logical problems. Details need not detain us here. 28

Yet, the point remains that the effect of singular denotation for "The kite" cannot be achieved without such context-relativization. With such relativization, the phrase "The kite" behaves almost 29 like demonstratives. Thus, given the elaborately specified context, "The kite is black" says nothing more than "That thing is black". Roughly, an utterance of sentences like "The kite is black" involves a story of the following sort: a unique object is somehow fixed in advance for the speaker and the audience; the utterance of "The" in the utterance of "The kite", then signifies that a member of the class denoted by "kite" has already been so fixed. Most 30 uses of "The kite", then, if we so prefer, are referential uses. Moreover, most uses of "The kite" are accompanied by an access to the object either with the object in the current field of perception or in the short-term memory. There is a variety of cases then where a definite description cannot be used to achieve a further reach. A referential use of "The kite" is linked to the non-Russellian character of "The kite" with or without speaker's reference.

The story of "The kite" is very far from the classical story that definite descriptions allow a speaker/audience to pick out a unique object 'unambiguously' solely by virtue of 'identifying descriptions', by virtue of the 'sense' of the predicates. Fortuitously or not, philosophers have traditionally chosen the following definite descriptions, among others: "The centre of mass of the Solar System", "The first line of Gray's Elegy", "The first man on the moon", "The author of Waverley", "The father of Charles II", "The inventor of bifocals", "The teacher of Aristotle", "The president of U.S. in 1980", "The husband of Nancy Reagan", "The present king of Sweden".

Intuitively, each of these denoting phrases pick out (i.e., enable an audience to pick out) a unique individual without a 'presentation' of the individual. As such, each of them allows a more or less 'further reach' such that singular denotation is achieved primarily via general background knowledge. Just as the paradigmatic use of "The kite" is referential, so the paradigmatic use of "The present king of Sweden" is attributive. The difference between the two paradigms is captured by the flexible parameter of 'further reach'. How is the 'further reach' accomplished in the supposedly Russellian definite descriptions?

A moment's reflection tells us that, for each item in the list, the singularity, i.e., the unambiguity, of denotation is facilitated by a special sort of "G"s. Thus, in a monarchy, there is just one king for a country at a time; in a monogamy, there is just one husband for a woman at a time; in a presidency, there is just one president for a country at a time; biologically, there is just one father for a person. Also, somewhat strenuously: usually, in earlier times, there used to be just one author of a book; usually, in earlier times, there used to be just one inventor (or, one known inventor anyway) for a particular instrument. How does "The teacher" in "The teacher of Aristotle" enable us to pick out a single person? Perhaps by "teacher" is meant "teacher who had the most durable influence" which picks out a unique individual for each historically significant person for us. "The teacher of Wittgenstein", for us philosophers, does not pick out one of his high-school teachers, but picks out Russell!

Much as the singularity of "The kite" is relativized to, roughly, contextual factors, the singularity of "The present king of Sweden" is relativized to what we might call, roughly, 'Information factors'. Thus, a user of paradigmatic attributive phrases has to master a lot of information regarding institutional, mathematical, scientific and other matters before she can figure out the singularity of such phrases. But, once she has mastered such information of rather larity of such phrases. But, once she has mastered such information of specific sorts, singularity becomes largely descriptional. The singularity of specific sorts, singularity becomes largely descriptional, not local, know-paradigmatic attributive descriptions is a matter of general, not local, know-ledge.

Descriptional singularity is a matter almost of language, if one's favourite view of language is Quinean. Quine identifies linguistic meaning with the widest of the community-wide beliefs; the wider the community, the more robust the belief; the more robust the belief, the more the belief approximates linguistic meaning. We manipulate, Quine suggests, the parameter of width, and, thus, in turn, divide the speech-community variously, according to various practical conveniences. When a certain community, so divided, gets established in its own right, the members of the community share a certain general knowledge³³ in terms of their descriptive vocabulary which they can use to achieve singular denotation in a general way. Yet, there is nothing 'necessary' or 'linguistic' about such sharing.

The singularity of "author of X" and of "inventor of Y", as I have already suggested, are quite tenuous in these days of group research and publication. "Teacher of Z" is even more tenuous, and to fix that we need a surprisingly small community—philosophers, for example. "Husband of X" is more tenuous now than fifty years ago, "father of Y" is less so. It is not that, for "husband of X" the relevant general knowledge is shared by a smaller community, but because the wider community is changing the relevant general knowledge. Even "king of Z" was non-singular in ancient Sparta. On the other hand, many supposedly non-Russellian predicates are frequently used with 'further reach'. "The car", "The stereo" in a family; "The dean", "The secretary" in a professional community; "The school", "The hospital" in a village community. (Notice that the paradigmatic uses of these phrases are referential with some 'further reach'.)

'Mundane' objects like tables, cups, kites, ashtrays and suitcases cannot usually be referred with a 'further reach' unless some of them are of outstanding interest to a certain group. So, when they are talked about, they usually need to be present before both the speaker and the audience. Perceptual access, in such cases, creates a tentative community, and perceptual knowledge the relevant knowledge shared by such a community. Perhaps, 'community' is not the proper term for such accidental gatherings. Let us, therefore, settle for 'group'.

ν

A group may be understood as any gathering of people using a language who share common knowledge of mutual interest of some portion of the world-real, imaginary or abstract; members of the group also share knowledge about the (potential) activities of each other. Groups are stable when the members of a group share a history of participation either in a pre-arranged institution (church, office, nation, research laboratory, etc.) or in social formations of their own (family, friends, clubs, reading circles, etc.). Or a group may be formed accidentally with the relevant portion of the world laid out in front of

them. Needless to say, an individual may participate in more than one group, and the extension of his predicates will 'chunk' out relevant portions of the world accordingly. When discussing an office-memo, "The president" is chunked to denote unambiguously a head of office; when debating foreign policy, "The president" is chunked to denote a head of state.

I shall continue to exemplify the notion of chunking by using "chunk". The intuitive idea is the following: predicative expressions in a language (typically, simple monadic predicates) are learnt by native speakers in accordance with something like Putnam's 'Principle of Social Division of Linguistic Labor'. The Predicates are chunked when a certain group stereotypically refers to items of mutual interest that fall within the range of a Putnamian predicate. Whether these items form a mere subset or a proper subset of the range of the predicate depends on the width and stability of the group concerned. Stereotypical reference, for the group, would usually be supplemented by all sorts of local collateral information. An empirically useful definition of "chunk" will have to await advances in psychological theory which would tell us how the uses of predicative expressions are accompanied by structured collateral information. The structured are accompanied by structured collateral information.

Another, closely similar, way of looking at the notion of chunking is as follows. Most predicative expressions, when used, contain explicit or implicit indexical elements. Chunking may be thought of as a common (cognitive) act of specifying these indexical elements in the same way by a group.

All (communicative) uses of language are performed with respect to a certain audience sharing common knowledge with the speaker. When the knowledge shared is by virtue of participation in *stable* groups, i.e., communities, the group concerned has a long 'further reach'; they can pick out the relevant object unambiguously whenever they are speaking among themselves in a wide variety of 'resource situations' just by virtue of their participation in the stable group.

Perhaps the farthest reach is provided by groups sharing mathematical knowledge. "Least prime" is possibly as 'thoroughly descriptional'36 and Russellian as one can get. There is just one way of chunking the ordered world of numbers. Among empirical phrases, superlatives work best when things can be ordered to map on to positive integers. "Shortest spy" works, because spies can be ordered in an order of magnitude to create the effect of "First spy when spies are arranged in an increasing magnitude of height" given that spy when spies are arranged in an increasing magnitude for the ordering of there are no duplications. Non-duplication is guaranteed for the ordering of numbers, not so guaranteed for spies or fish in the ocean. So, strictly, even "shortest spy" or "largest fish in the ocean" are not Russellian.

The social world elicits another kind of ordering in terms of heirarchies in the social institutions. Given a particular chunking, the structure of the institution is reflected in the predicates used by a stable group. Since in all

these situations the predicates themselves, relative to the chunking performed by the stable group concerned, effect singularity, should we say, as some authors have suggested recently,³⁸ that the uses of "The" preceding such 'Russellian' "G"s are superfluous?

Yet, such mathematical, scientific and institutional knowledges are one thing, knowledge of language quite another. Correct uses of "The" signify the knowledge of language of an English speaker. She uses it whenever she already shares with her audience the chunked reference of the predicate she is going to use next. A use of "The" signifies, across the board, that the speaker is in tune with the audience regarding the subject-matter of discourse, whatever be the current state of non-linguistic knowledge she shares with the audience. Uses of "The" take place in a gathering of same-G-chunk-knowers.

If a speaker does not yet share a knowledge with her audience, she first introduces the knowledge element—"A man came to my office today"—thus forming a local group and then uses the definite article: "The man was selling Encyclopaedias." A friend of mine walks into my office one morning and says: "The movie was great." If he is not speaking to himself, chances are that he told me the day before that he was going to see A Passage to India or something; otherwise, if he merely thought he had told me so, he invites my surprise—"I didn't know you went to a movie".

These examples have some further theoretical interests. I shall mention, very briefly, two such interests:

- (1) It is possible to notice here the dispensability of speaker's reference in favour of group reference even for anaphoric constructions. In the first example, an anaphoric use of "The man" is preceded by a securing of group reference; in the second, a speaker's merely 'having some object in mind' is not enough for successful communication. A choice between the notions of speaker's reference and group reference may not simply be a matter of taste.
- (2) If there is a feeling that the ensuing account is over-general, that all communication take place in the context of same-knowers, the first example, in particular, should help. Quantified expressions "A man", "All men", "Some men", are usually used to introduce knowledge and prospective referents; "The" is used to carry on with the knowledge so introduced. The real novelty, in an use of "The G is F" for an audience, is in "F". Here, then, is a glimpse of how quantified expressions are to be non-trivially marked off from definite descriptions pending detailed examination of some apparently obvious counter-examples.

We share a largely true picture of the world, as Donald Davidson has so powerfully emphasized in recent years, to be able to use language at all. That makes us general same-knowers. But we still need to cut that general picture down to various shapes for our referential needs. We still need to be

99

same-G-chunk-knowers. An use of "The", in my view, signals the achievement of that additional step.

Without getting into further details, the general picture should be reasonably clear by now. Paradigmatically, "The G" is used attributively in the context of stable and wide groups sharing a variety of general knowledge about chunking-relative unambiguous denotations. Russell's theory applies to a small and extreme fringe of such cases in accordance with Russell's interest in mathematical knowledge and his interest in the workings of a perfect language. Paradigmatically, "The G" is used referentially in the context of unstable and narrow local groups sharing a variety of short-term knowledge (either introduced by the speaker or available via perception). Human groups, however, are seldom absolutely stable or absolutely unstable. Given the structural variety of 'resource situations' in which human beings need to communicate with each other, these paradigmatic uses easily flow into each other, condensing at some points, thinning out at others but generally staying on the line of group reference.

So, just to recapitulate on this picture: "The least prime" is primarily used attributively; "The man over there" is primarily used referentially (even if someone said "The man over there, whoever he is"); "The winner of the Indianopolis race 500", "The murderer of Smith"—all these can be used both referentially and attributively since the referents of these phrases may either be within 'reach' or can be (mentally) located via structured knowledge about winners of races and rarity of gang murders. Given that our referential devices exploit the structure of our group-behaviour, we can think of the dual uses of "The G" as clustering in a middle area on the line of group reference and then gradually petering out to the extremes of single uses.42

Whatever is the reference (extension) of "G", "The G" always refers to the reference of "G" as chunked by a group. Group reference is, if we so prefer, an inherently pragmatic notion, and so is the notion of truth for an assertion of the form "The G is F". When a group is wide and stable enough to cover the entire population of English users, the group reference of "The G" may approximate, if we so prefer, the semantic reference of "The G". Semantic reference, then, is totalitarian group reference.43

The notion of group reference can, then, be used to give a unified account of the entire spectrum of definite descriptions from the most 'perfect' ones to the most 'imperfect' ones including the dual uses of some of them that cluster in the middle. The account is unified in that the uses of "The" do not vary; what varies is the group reference accompanying "The". Since uses of "The" do not vary, the account easily covers plural noun phrases; whether the relevant group reference concerns one "G" or many "G"s or exactly one "G" is to be read off from the particular "G" itself by the knowledgeable members of the group concerned.

Up to this point, this account has a significance for at least one aspect of the Russell-Strawson-Donnellan debate. In his original paper,44 Donnellan

complained that both Russell and Strawson tell only half of the story: Russell tells the non-referential (i.e., attributive) story, while Strawson tells the referential story. A complete story, for Donnellan, involves both. Further, for Donnellan, a complete story involves a sharp distinction. If I am correct, Strawson's insight can now be generalized as follows: all uses of definite descriptions are group referential; Russellian definite descriptions, if any, are just special cases of group reference under conditions quite external to linguistic knowledge of "The". Russell did not tell even half the story.

VI

The preceding account does not yet supply an account of what Donnellants thinks is the hallmark of referential uses. While, in our account, reference is achieved via the referent(s) fitting chunked "G"s, a referential use, for Donnellan, is independent of whether the referent fits "G" at all. Thus, in his classic example, someone might say (presumably pointing to a man over there), "The man over there drinking champagne is happy tonight", to which someone else might remark; "He is not drinking champagne, he is drinking water." Probably, no one in the 'vicinity' ('over there') is drinking champagne. So, no one, apparently, fits the chunked "man drinking champagne", yet the correct referent (signalled by the two uses of "he") is picked out by the audience.

This sort of example is somewhat different from classical examples like "The (present') king of France is bald" uttered when France is no longer a monarchy. To mark off this difference, I shall label Donnellan's example as a case of 'misdescription'—the description fails without a 'failure of reference'. The classical cases I shall label 'vacuous description'—reference 'fails' by virtue of the failure of description.

If we are not in the grip of some fairly strict notion of semantic reference, then it seems to me that examples of misdescription are of no theoretical consequence for an understanding of uses of "The". Donnellan observes46 correctly that 'a speaker means something by an utterance when he has a certain complex kind of intention involving recognition on the part of his audience of his intention'; it is simply not the case that anything goes. What goes depends on the circumstantial knowledge shared by the group. The very fact that an audience, in Donnellan's case, is able to pick out the referent testifies to some shared knowledge around "man over there drinking champagne". The shared knowledge concerns the chunked reference of "man over there". the pointing gesture, perceptual similarities between drinking champagne and drinking water, and so on. The case is obviously interesting from a psychological point of view: just how is the information contained in "champagne" overlooked for achieving reference? But this psychologically interesting question has nothing to do with the linguistic knowledge of uses of "The"; "The", as usual, signals an achievement of group reference.

Examples of vacuous descriptions, however, are a wholly different theoretical matter. For us, such cases are theoretically significant in at least two ways. First, according to our account, an use of "The" must be preceded by some shared knowledge. In an use of a vacuous description, however, there is simply no such knowledge to share. Feedows, we may be inclined to set up the (pragmatic?) truth-conditions for "The G is F" simply as follows: "The G is F" is true just in case the group referent(s) of "The G" satisfies "F"; it is false just in case the group referent of "The G" fails to satisfy "G". But we still need to specify a condition for the case in which "The G" does not have a group referent at all. Russell and Strawson address the problem as follows.

Suppose that France, at t, is not a monarchy. Suppose further that someone, in fact, says at t, 'with a perfectly serious air': "The king of France is wise" (call this sentence 'S'). How should we evaluate the truth-value of S as uttered by some one at t? Given the pre-theoretical agreement that S is a perfectly meaningful English sentence, Russell was pursuaded that both "The king of France is wise" and "The king of France is not wise" should have the value false without violating the law of excluded middle. This identification of the problem was then followed by the 'quantifier' interpretation of "The" and the subsequent scope distinctions. Strawson suggests that, since an utterance of S at t involves a 'reference failure', the question of S's truth-value does not arise.

Given our interest in the communicative function of language, we need not enter the Russell-Strawson debate right now; for we still have to make sense of the idea of someone uttering S at t 'with a perfectly serious air'. We cannot begin to appreciate the problem until further details about who refers (or fails to refer) with what background for whom are supplied. We need these details, since we do not assume that all language users necessarily participate in a homogenous group. Faced with the paucity of these details in the literature, we will have to strike out mostly on our own. I would suggest the following case to generate most of the interesting possibilities.

A native, X, has been brought up entirely under a monarchial system, and he has no idea about alternative political systems. X also knows how to identify the king of a country, e.g. when the king gives a public speech, the national flag is displayed behind him. In sum, all the (non-verbal) paraphernenational flag is displayed behind him. In sum, all the (non-verbal) paraphernelia that is usually associated with heads of states, in general, is also associated with X's idea of a king. While X was visiting U.S., he remained, due to acawith X's idea of a king. While X was visiting U.S., he remained, due to acawith X idea of a king. While X was visiting U.S., he remained, due to acawith X is idea of a king. While X was visiting U.S., he remained, due to acawith X is idea of a king. While X was visiting U.S., he remained, due to acawith X is idea of a king. While X was visiting U.S., he remained, due to acawith X is idea of a king. While X was visiting U.S., he remained, due to acawith X is idea of a king. While X was visiting U.S., he remained, due to acawith X is idea of a king. While X was visiting U.S., he remained, due to acawith X is idea of a king. While X was visiting U.S., he remained, due to acawith X is idea of a king. While X was visiting U.S., he remained is also associated with heads of states, in general, is also associated with heads of states, in general, is also associated with heads of states, in general, is also associated with heads of states, in general, is also associated with heads of states, in general, is also associated with heads of states, in general, is also associated with heads of states, in general, is also associated with heads of states, in general, is also associated with heads of states, in general, is also associated with heads of states, in general, is also associated with heads of states, in general, is also associated with heads of states, in general, is also associated with heads of states, in general, is also associated with heads of states, in general, is also associated wi

Before I do so, notice that there is absolutely no requirement in the case that the native's own language is non-English. It seems to me that a case

similar to the native's would obtain for most American pre-schoolers. Knowing that kings and queens are the only political entities a child ever encounters in her story books, we are likely to say to the child, while pointing at Ronald Reagan: "He is like a king, honey." Since the child does not understand what it is to be like a king and yet still not be a king, the child infers that she is looking at a king. So the case can easily be construed as a genuine case of homophonic indeterminacy due to differences in collateral information. The child, like the native, simply does not know and does not care that U.S. is not a monarchy.

Here, then, are the three personalities. Y is another native who has been to U.S. a few months before and is now jealous about the attention X is getting from the fellow-natives. It is likely, then, that Y would show off his knowledge of U.S. by contradicting X. LeRoy is a friend of X who knows a lot about X's country and the native's ways of thinking. LeRoy and X were together watching Ronald Reagan on T.V. last night. John Smith knows nothing about X or his country, and he was just passing by when X made that astonishing remark in plain English. How did they react individually? Ignoring the details of time and country, here is what happened.

Y, the other native, remarked: "I did not find him friendly towards the Russians at all" (Case 1). LeRoy decided to play along and said: "Well, I think you would change your opinion, if you listened to that one about bombing the Soviet Union in the next five minutes" (Case 2). John Smith stared at X for a moment and said; "I think you have got your politics mixed up, U.S. has never been a monarchy" (Case 3). Notice that all these variations in response obtained without varying the native's assertion, or, for that matter, without varying the native's 'mind'. The native is referring (or 'failing to refer') in the same way throughout.

Let me, first, discuss Case 2 only to set it aside, for the time being. How could LeRoy 'play along' when, for LeRoy there is no king of U.S.? LeRoy, obviously, is construing the use of "The king of U.S." as a referential use based on his short-term knowledge of last night. Moreover, given his general knowledge, he also knows that "The king of U.S." could be used attributively in the native land. Thus, despite the oddity of the native's utterance, there is no massive referential tension for LeRoy. Case 2, I shall assume, is similar to Donnellan's example of misdescription, and, therefore, we may ignore it.

As for Case 1, it is clear that Y has picked up the correct reference (signal-led by his use of "him") without any referential tension whatsoever. Moreover, he has picked up the correct reference both referentially and attributively. Thus, within the native dialect, he is surely contradicting X. This would be certified not only by LeRoy who agrees with Y, but also by other native listeners for whom "The king of U.S." has been used purely attributively. The other natives, I would think, would be justified in holding that one, not both, of X and Y is telling the truth. John Smith's Russellian verdict

that both X and Y are asserting something false plainly would not count, nor would John Smith's Strawsonian verdict that X and Y are asserting nothing at all. If, however, Russell and Strawson individuate language by dialects, then there is no problem.

In Case 3, John Smith has an access, if at all, to the reference of "The king of U.S." as uttered by X at t only attributively. Since he does not have any such access, Donnellan would suggest⁴⁸ that 'one or the other of the two views, Russell's or Strawson's, may be correct'. But why should John Smith's verdicts count in, say, Ohio, but not in the native land, given that the native has asserted in the same way in both cases? How should we explain the logical nightmare that the same native assertion changes truth values depending on where it is asserted? There is something wrong, then, with the (in particular, Strawson's) notion of 'reference failure' simpliciter.

However, if we take the audience seriously into account, i.e., if we generalize Strawson's notion of reference failure to failure of group reference, we may explain the cases even while keeping the native assertion 'fixed'. In Case 1, "The king of U.S." obviously has a group reference; hence the question of truth and falsity arose. In Case 3, there is still the native's reference of "The king of U.S.", but there is no group reference; hence the question of truth and falsity, apparently, does not arise. The question of truth and falsity does not arise for the perfectly good reason that a communication between X and John Smith never got started. In general, it seems to me, dialects can always be cut finely enough to render alleged cases of vacuous descriptions as cases of dialect transgression. Working now with a notion of 'truth in a dialect', we may not need to specify a third truth condition for vacuous descriptions.

However, moving away from the Russell-Strawson debate now, what importance should we attach to cases of dialect transgression? How would John Smith react, for example, to his own child when the child says at t; "The king of U.S. is old?" Would John Smith immediately embark on a lecture on the American politicals ystem? Or, would he stop the child saying: "I don't know what are you talking about?" Most parents, I would think, would do neither. If they cannot pick out the child's referent from local, contextual knowledge, they would start asking questions, search their memory, and, in general, try to find a group reference for "The king of U.S." under the assumption throughout that the child must be having a referent 'in mind'—a referent that must be publicly accessible somehow. Human communication, in general, is human. I doubt very much that there are descriptions which, even after all such tries, remain intrinsically vacuous when some one says something 'with a perfectly serious air.'49 I am unsure, therefore, about how much philosophical weight should be attached to alleged cases of vacuous descriptions.

Notes

*This is a revised and enlarged version of papers presented earlier at the University of Toronto (October 1985) and at the Pacific Division Meeting of the American Philosophical Association at Los Angeles (March 1986). My thanks to Nicholas Griffin and Banks Tapscott who commented on these occasions. Thanks also to Angus Kerr-Lawson, Bernard Linsky, Robert Martin and David Kaplan for many helpful suggestions. Michael Webster kept a friendly critical watch throughout.

- Saul Kripke, 'Speaker's Reference and Semantic Reference', in French, Uehling and Wettstein (eds.), Contemporary Perspectives in the Philosophy of Language, Minneapolis, University of Minnesota Press, 1979, p. 22.
- 2. Except the standard restriction to definite descriptions occurring in the subject position of a simple subject-predicate sentence. In general, I shall not be concerned with definite descriptions in the predicate position. Another, fairly standard restriction would be added later. See note 47.
- 3. I am ignoring the possibility that some later uses of "The" might have been influenced by Russell's theory, especially in the philosophical community, e.g. 'Why is the free will problem so persistent? Partly, I suspect, because it is called the free will problem' (Daniel Dennett, Brainstorms, Bradford Books, 1978, p. 286).
- 4. Some of the empirical conclusions reached by me, e.g. the theory of a 'continuum' of shared knowledge (Section V) was also reached by some linguists (e.g. Geoffrey Leech, Semantics, Penguin Books, 1981, pp. 156-58). I find no indication, however, that Leech appreciates the significance of these conclusions for Russell's theory. Leech's use of the notion of 'uniqueness' is particularly confusing.
- 5. Double quotes are used throughout as mention-quotes. Single quotes are used for citation, and for ironical, technical and other non-standard uses.
- 6. Saul Kripke, op. cit., p. 16.
- "Referent" here may be construed as "referent simpliciter". Possibly what Russell 'had in mind' was semantic referent. But to build that into the statement of the theory itself, as Kripke does, is to prejudge a highly contentious issue. More of this later.
- 8. P.F. Strawson, 'On Referring' in Mind, July 1950; David Kaplan, 'What is Russell's Theory of Descriptions' in Yourgrau and Breck (eds) Physics, Logic and History, New York, Plenum Press, 1970.
- 9. Cf. Gareth Evans, The Varieties of Reference, Oxford, Oxford University Press, 1982, pp. 51-60, for a recent defence of Russell's theory from this point of view. See also, George Wilson, 'On Definite and Indefinite Descriptions' in The Philosophical Review, 87, 1978, pp. 48-76.
- 10. I am ignoring Montague's curious suggestion of attaching, in an Intensional Logic, an independent meaning to "The" via Russell's Strong Theory. Cf. Dowty, Hall and Peters, Introduction to Montague Semantics, Dordrecht, Holland, D. Reidel Publishing Co., 1981, p. 195.
- 11. Noam Chomsky, Essays on Form and Interpretation, New York, North-Holland, 1977, pp. 47-50.
- 12. Keith Donnellan, 'Reference and Definite Descriptions' in The Philosophical Review, 75, 1966.
- 13. These examples are from William Martin, 'A Logical Form Based on the Structural Description of Events' in Vaina and Hintikka (eds), Cognitive Constraints on Communication, Dordrecht, Holland, D. Reidel Publishing Co., 1984, p. 214.
- 14. Kripke, op. cit., pp. 13-21.
- 15. The notion of speaker's reference was extensively used by Donnellan in his original paper. Kripke's contribution lies in showing that this notion, possibly, is a pragmatic one. I shall not discuss this controversial notion in this paper. But I shall throw enough

hints at several places to suggest that this notion, along with the complimentary notion

- Variations on this general theme are: primary/secondary reference (John Searle, 'Referential and Attributive' in Expression and Meaning, Cambridge, Cambridge University Press, 1979, pp. 137-61); effective/non-effective reference (D.E. Over, 'Effective and Non-effective Reference' in Analysis, Vol. 43, No. 2, March 1983, pp. 85-91); designational and non-designational reference (M. Devitt, Designation, New York, Columbia University Press, pp. 36-41), etc.
- 17. Kripke, op. cit., p. 6, p. 22.
- Alternatively, 'indefinite definite descriptions' (Donnellan), 'indeterminate descrip-
- 19. Kripke, op.cit.
- This line has been vigorously pursued informally in Howard K. Wettstein, 'Demonstrative Reference and Definite Descriptions' in Philosophical Studies, 40, 1981, pp. 241-57; more formally, in David Kaplan, 'Dthat' and 'On The Logic of Demonstratives' in French, Uehling & Wettstein (eds.), op. cit.
- See also note 29 below.
- 22. Strawson, op. cit., p. 341.
- 23. Bertrand Russell, 'On Denoting', in Mind, 1905.
- 24. Imagine this paper being verbally presented to an audience.
- Jon Barwise and John Perry, Situations and Attitudes, Cambridge, The M.I.T. Press, 25.
- Which is persuasively challenged in Wilson, op. cit., pp. 48-53. 26.
- Wettstein, op. cit., pp. 246-47. 27.
- 28. See, Kaplan, op. cit.
- This qualification is important. "The table" and "That table" may have similar uses, but it requires a lot of arguments and evidence to postulate redundancy in a natural system. In any case, Fitch ('Indeterminate Descriptions' in The Canadian Journal of Philosophy, June 1984) has argued that there are subtle differences in the uses of "The table" and "That table", even though both are, in a broad sense, contextdependent.
- 30. "Most", not "all'. Some authors (e.g. John Pollock, Language and Thought, Princeton, N.J., Princeton University Press, 1982, p. 108) have held mistakenly that an 'improper' definite description, if successful, can only be used referentially. Knowing that James frequently watches late night movies on T.V. on Sundays and knowing further that he is, as usual, late for work this Monday morning, I can say to his frustrated boss, in an attempt to save James' neck, "The movie James was watching must have been absorbing", without having the faintest idea of what movie James was watching last night. Of course, I would say this to an audience (i.e. the boss, in the present case) who shares similar knowledge about James' habits—that is the central point of this paper; this use of "The movie" is attributive, nonetheless. Linsky's example, "The table is the most important article of furniture in a dining room" (Leonard Linsky, Referring, London, Routledge & Kegan Paul, 1967, p. 63), however, won't do; "table", in Linsky's example, is used as a 'species' term, not a 'sortal' term.
- 31. I am ignoring here the complications generated by the anaphoric uses of "The table" which, in turn, seem to be linked to the phenomenon of speaker's reference. Cf. Keith Donnellan, 'Speaker's Reference, Descriptions and Anaphora' in French, Uehling and Wettstein (eds), op. cit., Again, if the ensuing account is correct, a general theory of definite descriptions may not require speaker's reference as a theoretical construct.
- 32. Cf. W.V.O. Quine, 'Reply to Chomsky' in Davidson and Hintikka (eds) Words and Objections, 1969, p. 310.
- Knowledge, not object(s). Here I depart from the so-called "Locational" or "Shared Object" theory of descriptions proposed by Hawkins (John Hawkins, 'On Surface Definite Articles in English' in Van Der Auwera (ed.), The Semantics of Determiners,

Baltimore, University Park Press, 1980). Hawkin's theory has been critically discussed by, among others, Christopher Lyons ('The Meaning of English Definite Articles', in Van Der Auwera (ed.), op. cit.) and Thrane (Torben Thrane, Referential Semantic Analysis, Cambridge, Cambridge University Press, 1980, p. 185-89). I believe the ensuing theory is immune from such criticisms.

- Hilary Putnam, 'The Meaning of "Meaning" in Gunderson (ed.) Minnesota Studies in the Philosophy of Science, Vol. 7, Minneapolis, University of Minnesota Press, 1975.
- 35. This notion has close parallels with the notion of chunking employed in work in Artificial Intelligence.
- 36. Cf. Nathan Salmon, Reference and Essence, Princeton, N.J., Princeton University Press, 1981, pp. 14-22.
- 37. Which makes it possible for us to say "One of the fastest runners...", but not "One of the smallest positive integers...".
- 38. See, Ruth Millikan, Language, Thought and Other Biological Categories, Cambridge, The M.I.T. Press, 1984, pp. 175-91. Millikan's suggestion thus precipitates yet another theoretically uneasy distinction between superfluous and non-superfluous uses of "The". Nevertheless, Millikan's discussion provides one of the most thorough taxonomies of the uses of "The".
- 39. Strawson (Strawson, op. cit., p. 335) suggested that referring terms are used to forestall the question: 'Who/what are you talking about?' This suggestion is developed by Hawkins (Definiteness and Indefiniteness, London, Croom & Helm, 1978) in his 'Locational' theory of definite descriptions. If the preceding account is correct, then, so far as definite descriptions are concerned, a use does not forestall the question; a use signifies that the question has already been forestalled, for the group, prior to the use. Strawson's thoughts on this point, it seems to me, have changed somewhat subsequently. Thus, in 'Singular Terms and Predication' in The Journal of Philosophy, 58, July 1961, pp. 399-402, Strawson stresses that the 'identificatory function' of a singular term must involve 'the thought of some object already within the reach of the hearer's own knowledge, experience, or perception'. For an interesting discussion of this point, David S. Schwarz, Naming and Referring, Berlin, Walter de Gruyter, 1979, pp. 38-99.
- 40. Which suggests the possibility that, if the predicates of a language come chunked (possibly in a small and closed community where all activities are done in a single group), a definite article would be functionally superfluous. Presumably, such a community would still require the idiom "exactly one", e.g. in answer to the question, "How many arrows hit zombies?" Russell's analysis of "exactly one arrow hit zombies" would still apply without there being definite descriptions in the language! A similar point is interestingly discussed in Millikan, op. cit., pp. 184-88. Of related interest is the discussion of 'non-present reference' in the American Sign Language in Ursula Bellugi and E. Klima, 'From Gesture to Sign: Deixis in a Visual-Gestural Language' in Jarvella and Klein (eds), Speech, Place and Action, Chichester, John Wiley, 1982, p. 301.
- 41. See, for example, Donald Davidson, 'On the Very Idea of a Conceptual Scheme' in *Inquiries into Truth and Interpretation*, Oxford, Clarendon Press, 1984, pp. 183-98.
- 42. It is surprising that authors, who emphasize the fundamental roles played by the notions of group and mutually shared beliefs in our understanding of communicative functions of language, nevertheless insist on a strict separability between utterer's meaning and semantic meaning to explain Donnellan's distinction. I have in mind authors such as Kent Bach and Robert M. Harnish, Linguistic Communication and Speech Acts, Cambridge, The M.I.T. Press, 1979, especially p. 287. However, Bach and Harnish's theory of 'communicative inference' does seem to provide a better understanding of the sort of cases discussed in Section VI, though I am unsure about the relevance of this theory for "The". Harnish's recent paper ("Communicative Inference: An Inferential Model' in Conceptus, 18, 1984) came too late in my hands for a careful study for this paper.

- Replace "speaker's reference" with "group reference" and compare: 'a diachronic Replace specified and compare: 'a diachronic account of the evolution of language is likely to suggest that what was originally a mere speaker's reference may, if it becomes habitual in a community, evolve into 'semantic reference' (Saul Kripke, op. cit., p. 22). I find the idea of a speaker's reference becoming 'habitual in a community' incoherent; how can the rest of the community have an access to what a speaker 'has in mind'? Donnellan, 'Reference and Definite Descriptions', op. cit.
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- Donnellan, 'Reference and Definite Descriptions', op. cit., pp. 283, 285; Donnellan, Putting Humpty Dumpty Together Again' in The Philosophical Review, 77, April
- Donnellan, 'Putting Humpty Dumpty Together Again', op. cit., pp. 212-14.
- Apart, possibly, from the knowledge that "G" does not have a chunked reference. Why then is "The G" used in the first place except, perhaps, for asserting "The G does not exist"? But, then, what is the content of this last assertion in terms of our account? Like Donnellan ('Reference and Definite Descriptions', op. cit., p. 284), I shall set aside such problematic cases from the present discussion, simply because such uses of "The G" are not referential at all.
- 48. Donnellan, 'Reference and Definite Descriptions', op. cit., p. 283.
- 49. Except, perhaps, for cases like "The largest positive integer must be greater than a trillion". For most ordinary, non-mathematicians who cannot construct the relevant reductio proof, the description, I would think, is non-vacuous. Once we know the proof, however, I do not see how to achieve group reference with, for example, a child in the third grade. The only alternative, for the audience, in such global and purely abstract contexts, is either to change the subject or, indeed, to emabark on a lecture on number theory.